

HONORS COLLEGE ALGEBRA

ALGEBRA REVIEW MATERIAL

ALGEBRA REVIEW: Sets of Numbers

(Textbook Pages: Pages 2 – 5)

A) Define each of the following Sets of Numbers.

1) Natural Numbers (N)

The counting numbers: $\{1, 2, 3, 4, 5, \dots\}$

2) Whole Numbers (W)

The natural numbers and ZERO: $\{0, 1, 2, 3, 4, 5, \dots\}$

3) Integers (I)

The whole numbers and the natural numbers opposites: $\{\dots -3, -2, -1, 0, 1, 2, 3 \dots\}$

4) Rational Numbers (Q)

A number that can be expressed as a fraction p/q where p and q are integers with $q \neq 0$

The set of all decimals that repeat or terminate

Match the various forms in which a rational number may be written to the example given:

(Mixed Number, Whole Number, Integer, Natural Number, Terminating Decimal, Repeating Decimal)

Example	Form	Example	Form	Example	Form
5	<i>Natural Number</i>	$\frac{4}{7}$	<i>Fraction</i>	9.2111	<i>Terminating Decimal</i>
0	<i>Whole Number</i>				
		$7\frac{8}{11}$	<i>Mixed Number</i>	$9.\overline{21}$	<i>Repeating Decimal</i>
-5	<i>Integer</i>				

5) Irrational Numbers (Ir)

A number that cannot be expressed as a fraction p/q where p and q are integers with $q \neq 0$

Non-terminating, non-repeating decimals

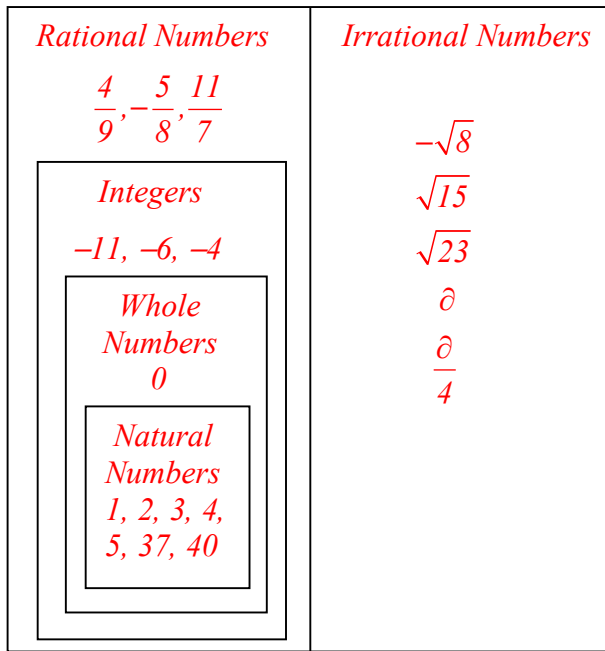
6) Real Numbers (R)

The rational numbers and the irrational numbers

The numbers corresponding to points on the number line

The set of all decimals

B) Copy the VENN DIAGRAM of Real Numbers (Figure 6) from page 5 of the textbook:



C) State the most specific set of numbers each of the following sets represent. Use symbols not words.

_____ *I* 1) $\left\{-3, \frac{16}{4}, -\sqrt{25}, 0, 12\right\}$

_____ *Q* 2) $\left\{-\sqrt{64}, 7\frac{2}{5}, -9.55, \frac{13}{72}, 4.\overline{1237}, 5, .72\overline{3}\right\}$

_____ *N* 3) $\left\{8, \frac{16}{4}, \sqrt{49}, 12, 3^2\right\}$

_____ *R* 4) $\left\{-\sqrt{64}, 7\frac{2}{5}, 7.121121112..., \frac{13}{72}, 4.\overline{1237}, \sqrt{12}, .72\overline{3}\right\}$

_____ *W* 5) $\left\{8, \frac{16}{4}, 0, \sqrt{49}, 12, 3^2\right\}$

_____ *Ir* 6) $\left\{\sqrt{12}, 7.121121112..., \frac{7\pi}{4}\right\}$

D) Exercises: page 12 #1 – 14

1. *A, B, C, D, F* 2. *B, C, D, F* 3. *D, F* 4. *A, B, C, D, F* 5. *E, F* 6. *D, F*

7. *A rational number CAN be written as a fraction; an irrational numbers CANNOT be written as a fraction.*

8. *These values are ESTIMATES for π not actually π .*

9. *Any fraction, terminating or repeating decimal that does not reduce to an integer*

10. *Any negative integer*

11. *1, 3*

12. *0, 1, 3*

13. *$-6, \frac{-12}{4}, 0, 1, 3$*

14. *$-6, \frac{-12}{4}, \frac{-5}{8}, 0, \frac{1}{4}, 1, 3$*

ALGEBRA REVIEW: Properties of Real Numbers

(Textbook Pages: Pages 8 – 11)

- A) Complete the equation to illustrate the following Properties of Real Numbers.
B) **Describe** the property with key words or phrases. **DO NOT** copy the definitions (p. 11) from the book.

$$\begin{array}{l} 1A) \text{ Closure } + \quad \} \quad 4 + 6 = 10 \quad \underline{10 \text{ is a real number}} \\ \text{Closure } \times \quad \} \quad 4 \cdot 6 = 24 \quad \underline{24 \text{ is a real number}} \end{array}$$

B) *When real numbers are added or multiplied the answer is still a real number.*

$$\begin{array}{l} 2A) \text{ Commutative } + \quad \} \quad 2 \cdot (4 + 6) = 2 \cdot (6 + 4) \\ \text{Commutative } \times \quad \} \quad 2 \cdot (4 + 6) = (4 + 6) \cdot 2 \end{array}$$

B) *The **ORDER** in which you add or multiply numbers does not change the answer.*

$$\begin{array}{l} 3A) \text{ Associative } + \quad \} \quad 2 + (4 + 6) = (2 + 4) + 6 \\ \text{Associative } \times \quad \} \quad 2 \cdot (4 \cdot 6) = (2 \cdot 4) \cdot 6 \end{array}$$

B) *The way numbers that are being added or multiplied are **GROUPED** does not change the answer.*

$$4A) \text{ Identity } + \quad 2 \cdot (4 + 0) = 2 \cdot 4$$

B) *When you **ADD ZERO** to a number, the answer is that identical number.*

$$5A) \text{ Identity } \times \quad 2 \cdot (4 \cdot 1) = 2 \cdot 4$$

B) *When you **MULTIPLY** a number **BY ONE**, the answer is that identical number.*

$$6A) \text{ Inverse } + \quad 2 \cdot (4 + (-4)) = 2 \cdot 0$$

B) *When you **ADD** a number and it's **OPPOSITE**, the answer is ZERO, the identity for addition.*

$$7A) \text{ Inverse } \times \quad 2 \cdot (4 \cdot \frac{1}{4}) = 2 \cdot 1$$

B) *When you **MULTIPLY** a number and its **RECIPROCAL**, the answer is ONE, the identity for multiplication.*

$$8A) \text{ Distributive } \quad 2 \cdot (4 + 6) = 2 \cdot 4 + 2 \cdot 6$$

(x over +)

B) *Multiplication can be distributed over addition.*

The product of a number and the sum of two numbers is the same as the sum of the products of the number and each addend. The number is multiplied by each addend and then the products are added.

- C) State the name of the property illustrated by each equation, be sure to include the operation. Behind each give a reason that supports your answer (i.e. order of multiplication changed).

Commutative Addition 1) $9 + (\sqrt{5} + 0) = 9 + (0 + \sqrt{5})$ 0 and the $\sqrt{5}$ changed **ORDER** (they were added)

Associative Multiplication 2) $2 + 7\left(\frac{2}{3} \cdot \frac{3}{2}\right) = 2 + \left(7 \cdot \frac{2}{3}\right) \cdot \frac{3}{2}$ The **GROUPING** of $7, 2/3$, and $3/2$ was changed (they were multiplied)

Identity Multiplication 3) $2 + 7 \cdot 1 = 2 + 7$ 7 was **MULTIPLIED BY ONE** to get the identical number, 7

Inverse Addition 4) $6(-4 + 4) = 6(0)$ 4, the **OPPOSITE** of -4 , was **ADDED** to -4 to get zero

Distributive (x over +) 5) $6(-4 + 4) = -24 + 24$ 6 was **MULTIPLIED** by the addends -4 and 4 , then those products were **ADDED**

Identity Addition 6) $9 + (\sqrt{5} + 0) = 9 + \sqrt{5}$ **ZERO** was **ADDED** to $\sqrt{5}$ to get the identical number, $\sqrt{5}$

Inverse Multiplication 7) $2 + 7\left(\frac{2}{3} \cdot \frac{3}{2}\right) = 2 + 7 \cdot 1$ $2/3$ was **MULTIPLIED BY** $3/2$, its **RECIPROCAL**, to get 1

- D) Exercises: page 14 #51 – 66

For #63 – 66 Show work (distribution and reducing of fractions) --- DO NOT use a calculator.

51. Distributive (x over +)

52. Commutative Multiplication

53. Inverse Multiplication

54. Inverse Multiplication

55. Identity Addition

56. Closure Additon

57. No $2 - 4 = -2$ but $4 - 2 = 2$

58. No $(4 - 2) - 1 = 1$ but $4 - (2 - 1) = 3$

59. $p(8 - 14) = -6p$

60. $x(15 - 10) = 5x$

61. $-3z + 3y$

62. $-2m - 2n$

63. $\frac{10}{11}(22z) = \frac{10}{1}(2z) = 20z$

64. $\left(\frac{3}{4}r\right)(-12) = \left(\frac{3}{1}r\right)(-3) = -9r$

65. $-\frac{1}{4}(20m + 8y - 32z) =$

66. $\frac{3}{8}\left(\frac{16}{9}y + \frac{32}{27}z - \frac{40}{9}\right) = \frac{3}{8}\left(\frac{16}{9}y\right) + \frac{3}{8}\left(\frac{32}{27}z\right) - \frac{3}{8}\left(\frac{40}{9}\right)$

$-\frac{1}{4}(20m) - \frac{1}{4}(8y) - \frac{1}{4}(-32z)$
 $-1(5m) - 1(2y) - 1(-8z)$
 $-5m - 2y + 8z$

Always reduce
before multiplying –
Use cancellation

$1\left(\frac{2}{3}y\right) - 1\left(\frac{4}{9}z\right) - 1\left(\frac{5}{3}\right) = \frac{2}{3}y + \frac{4}{9}z - \frac{5}{3}$

Exercises: page 16 #85 – 88 (Rewrite the problem to indicate the mental math you are doing.)

85. $(72 + 28)17 = (100)(17) = 1700$

86. $32(80 + 20) = 32(100) = 3200$

87. $\left(123\frac{5}{8} - 23\frac{5}{8}\right)1\frac{1}{2} =$

88. $17\frac{2}{5}\left(14\frac{3}{4} - 4\frac{3}{4}\right) =$

$100\left(1\frac{1}{2}\right) = 100(1.5) = 150$
 $100\left(1\frac{1}{2}\right) =$
 $100(1) + 100\left(\frac{1}{2}\right)$

$17\frac{2}{5}(10) = \frac{87}{5}(10) =$

ALGEBRA REVIEW: Order of Operations

(Textbook Pages: Pages 5– 7)

A) Complete the following using the order of operations. Do only **ONE STEP** at a time.

For each step, list the specific problem you are doing for that step and its answer in the first column, and then substitute that answer into the problem and state the result in the second column.

The first one is completed as an example.

$$1) \quad 5 - 7 + 3^3 \div 9 \cdot (7 - 9)$$

a) Problem/Answer: $7 - 9 = -2$ Result: $5 - 7 + 3^3 \div 9 \cdot (-2)$

b) Problem/Answer: $3^3 = 27$ Result: $5 - 7 + 27 \div 9 \cdot (-2)$

c) Problem/Answer: $27 \div 9 = 3$ Result: $5 - 7 + 3 \cdot (-2)$

d) Problem/Answer: $3 \cdot (-2) = -6$ Result: $5 - 7 + (-6)$

e) Problem/Answer: $5 - 7 = -2$ Result: $(-2) + (-6)$

f) Problem/Answer: $(-2) + (-6) = -8$ Result: -8

$$2) \quad 8 + (-3^2 + 3) \div 2 \cdot 4 - 6$$

$$3) \quad [24 \div (1 - 3)^2 + 3 \cdot (-2)]^4$$

Problem/Answer	Result
a) $-3^2 = -9$	$8 + (-9 + 3) \div 2 \cdot 4 - 6$
b) $-9 + 3 = -6$	$8 + (-6) \div 2 \cdot 4 - 6$
c) $-6 \div 2 = -3$	$8 + (-3) \cdot 4 - 6$
d) $-3 \cdot 4 = -12$	$8 + (-12) - 6$
e) $8 + (-12) = -4$	$-4 - 6$
f) $-4 - 6 = -10$	-10

Problem/Answer	Result
a) $1 - 3 = -2$	$[24 \div (-2)^2 + 3 \cdot (-2)]^4$
b) $(-2)^2 = 4$	$[24 \div 4 + 3 \cdot (-2)]^4$
c) $24 \div 4 = 6$	$[6 + 3 \cdot (-2)]^4$
d) $3 \cdot (-2) = -6$	$[6 + (-6)]^4$
e) $6 + (-6) = 0$	0^4
f) $0^4 = 0$	0

$$4) \quad [2 + (3 - 5)6] \div (5 \cdot 8 - 10)$$

Problem/Answer	Result
a) $3 - 5 = -2$	$[2 + (-2)6] \div (5 \cdot 8 - 10)$
b) $(-2)(6) = -12$	$[2 + (-12)] \div (5 \cdot 8 - 10)$
c) $2 + (-12) = -10$	$-10 \div (5 \cdot 8 - 10)$

Problem/Answer	Result
d) $5 \cdot 8 = 40$	$-10 \div (40 - 10)$
e) $40 - 10 = 30$	$-10 \div 30$
f) $-10 \div 30 = -1/3$	$-\frac{1}{3}$

$$5) \quad 12 + 25 \div (2 + 3) \cdot (4 - 5)^2$$

$$6) \quad -2^2 [4(8 - 2 \cdot 3) + 7] - 2^2$$

Problem/Answer	Result
a) $2 + 3 = 5$	$12 + 25 \div 5 \cdot (4 - 5)^2$
b) $4 - 5 = -1$	$12 + 25 \div 5 \cdot (-1)^2$
c) $(-1)^2 = 1$	$12 + 25 \div 5 \cdot 1$
d) $25 \div 5 = 5$	$12 + 5 \cdot 1$
e) $5(1) = 5$	$12 + 5$
f) $12 + 5 = 17$	17

Problem/Answer	Result
a) $2 \cdot 3 = 6$	$-2^2 [4(8 - 6) + 7] - 2^2$
b) $8 - 6 = 2$	$-2^2 [4(2) + 7] - 2^2$
c) $4(2) = 8$	$-2^2 [8 + 7] - 2^2$
d) $8 + 7 = 15$	$-2^2 (15) - 2^2$
e) $-2^2 = -4$	$-4(15) - 2^2$
f) $2^2 = 4$	$-4(15) - 4$
g) $-4(15) = -60$	$-60 - 4$
h) $-60 - 4 = -64$	-64

$$7) \quad \frac{(3 - 5 \cdot 2^3 + 1) \div 9}{(5 - 6)^3 + 6 \div 2}$$

Numerator	
Problem/Answer	Result
a) $2^3 = 8$	$(3 - 5 \cdot 8 + 1) \div 9$
b) $5 \cdot 8 = 40$	$(3 - 40 + 1) \div 9$
c) $3 - 40 = -37$	$(-37 + 1) \div 9$
d) $-37 + 1 = -36$	$-36 \div 9$
e) $-36 \div 9 = -4$	-4

Denominator	
Problem/Answer	Result
a) $5 - 6 = -1$	$(-1)^3 + 6 \div 2$
b) $(-1)^3 = -1$	$-1 + 6 \div 2$
c) $6 \div 2 = 3$	$-1 + 3$
d) $-1 + 3 = 2$	2

FINAL: Problem/Answer $\frac{-4}{2} = -2$

B) Exercises: page 12 #15 – 42 Show **ALL** work

For problems #15-21 show the factors you are multiplying and the solution

$$15. \quad 3 \times 3 \times 3 \times 3 = 81$$

$$16. \quad -(3 \times 3 \times 3 \times 3 \times 3) = -243$$

$$17. \quad -(2 \times 2 \times 2 \times 2 \times 2 \times 2) = -64$$

$$18. \quad (-3) \times (-3) \times (-3) \times (-3) = 81$$

$$19. \quad (-2) \times (-2) \times (-2) \times (-2) \times (-2) = -32$$

$$20. \quad (-3) \times (-3) \times (-3) \times (-3) \times (-3) = -243$$

$$21. \quad (-3) \times (-3) \times (-3) \times (-3) \times (-3) \times (-3) = 729$$

$$22. \quad \text{negative} \dots \text{positive}$$

$$23. \quad -5^2 = -(5 \times 5) = -25$$

$$24. \quad -(-5)^3 - (-5)^2 =$$

$$(-5)^2 = (-5) \times (-5) = 25$$

$$- [(-5) \times (-5) \times (-5)] - [(-5) \times (-5)]$$

$$- (-5)^2 = - [(-5) \times (-5)] = -(25) = -25$$

$$-(-125) - 25 = 100$$

B) Exercises: page 12 #15 – 42 Show **ALL** work (Continued)

For problems 25-42 copy the problem and show all steps

All problems should be completed WITHOUT a calculator

Fraction work should be shown (LCD, Reducing) --- DO NOT use a calculator.

$$\begin{aligned} 25. \quad & 8^2 - (-4) + 11 \\ & 64 + 4 + 11 \\ & 68 + 11 = 79 \end{aligned}$$

$$\begin{aligned} 26. \quad & 16(-9) - 4 \\ & -144 - 4 = -148 \end{aligned}$$

$$\begin{aligned} 27. \quad & -2 \cdot 5 + 12 \div 3 \\ & -10 + 12 \div 3 \\ & -10 + 4 = -6 \end{aligned}$$

$$\begin{aligned} 28. \quad & 9 \cdot 3 - 16 \div 4 \\ & 27 - 16 \div 4 \\ & 27 - 4 = 23 \end{aligned}$$

$$\begin{aligned} 29. \quad & -4(9 - 8) + (-7)(2)^3 \\ & -4(1) + (-7)(2)^3 \\ & -4(1) + (-7)(8) \\ & -4(1) + (-56) = -60 \end{aligned}$$

$$\begin{aligned} 30. \quad & 6(-5) - (-3)(2)^4 \\ & -30 - (-3)(2)^4 \\ & -30 - (-3)(16) \\ & -30 - (-48) = -30 + 48 = 18 \end{aligned}$$

$$\begin{aligned} 31. \quad & (4 - 2^3)(-2 + \sqrt{25}) \\ & (4 - 8)(-2 + 5) \\ & (-4)(3) = -12 \end{aligned}$$

$$\begin{aligned} 32. \quad & [-3^2 - (-2)][\sqrt{16} - 2^3] \\ & [-9 - (-2)][4 - 8] \\ & (-9 + 2)(-4) \\ & (-7)(-4) = 28 \end{aligned}$$

$$\begin{aligned} 33. \quad & \left(-\frac{2}{9} - \frac{1}{4}\right) - \left[-\frac{5}{18} - \left(-\frac{1}{2}\right)\right] \\ & \left(-\frac{8}{36} - \frac{9}{36}\right) - \left[-\frac{5}{18} - \left(-\frac{9}{18}\right)\right] \\ & \left(-\frac{17}{36}\right) - \left[-\frac{5}{18} + \frac{9}{18}\right] \\ & \left(-\frac{17}{36}\right) - \left[\frac{4}{18}\right] \\ & \left(-\frac{17}{36}\right) - \left[\frac{8}{36}\right] = -\frac{25}{36} \end{aligned}$$

$$\begin{aligned} 34. \quad & \left[-\frac{5}{8} - \left(-\frac{2}{5}\right)\right] - \left(\frac{3}{2} - \frac{11}{10}\right) \\ & \left[-\frac{25}{40} - \left(-\frac{16}{40}\right)\right] - \left(\frac{15}{10} - \frac{11}{10}\right) \\ & \left[-\frac{9}{40}\right] - \left(\frac{4}{10}\right) \\ & \left[-\frac{9}{40}\right] - \left(\frac{16}{40}\right) = -\frac{25}{40} = -\frac{5}{8} \end{aligned}$$

$$\begin{aligned} 35. \quad & \frac{-8 + (-4)(-6) \div 12}{4 - (-3)} \\ & \frac{-8 + 24 \div 12}{4 + 3} \\ & \frac{-8 + 2}{7} = \frac{-6}{7} \end{aligned}$$

$$\begin{aligned} 36. \quad & \frac{15 \div 5 \cdot 4 \div 6 - 8}{-6 - (-5) - 8 \div 2} \\ & \frac{3 \cdot 4 \div 6 - 8}{-6 - (-5) - 4} \\ & \frac{12 \div 6 - 8}{-6 + 5 - 4} \\ & \frac{2 - 8}{-1 - 4} = \frac{-6}{-5} = \frac{6}{5} \end{aligned}$$

$$37. 2(q - r)$$

$$2(8 - (-10))$$

$$2(8 + 10)$$

$$2(18) = 36$$

$$38. \frac{p}{q} + \frac{3}{r}$$

$$\frac{-4}{8} + \frac{3}{-10}$$

$$\frac{-1}{2} + \frac{3}{-10}$$

$$-\frac{5}{10} - \frac{3}{10} = -\frac{8}{10} = -\frac{4}{5}$$

$$39. \frac{q+r}{q+p}$$

$$\frac{8+(-10)}{8+(-4)}$$

$$\frac{8+(-4)}{8+(-4)}$$

$$\frac{-2}{4} = -\frac{1}{2}$$

$$40. \frac{3q}{3p-2r}$$

$$\frac{3(8)}{3(-4) - 2(-10)}$$

$$\frac{24}{-12 - (-20)}$$

$$\frac{24}{-12 + 20} = \frac{24}{8} = 3$$

$$41. \frac{3q}{r} - \frac{5}{p}$$

$$\frac{3(8)}{-10} - \frac{5}{-4}$$

$$\frac{24}{-10} + \frac{5}{4}$$

$$\frac{12}{-5} + \frac{5}{4}$$

$$-\frac{48}{20} + \frac{25}{20} = -\frac{23}{20}$$

$$42. \frac{\frac{q}{4} - \frac{r}{5}}{\frac{p}{2} + \frac{q}{2}}$$

$$\frac{\frac{8}{4} - \frac{-10}{5}}{\frac{4}{2} + \frac{5}{2}}$$

$$\frac{-4 + 8}{2 + 2}$$

$$\frac{2 - (-2)}{-2 + 4} = \frac{4}{2} = 2$$

ALGEBRA REVIEW: Rules of Exponents

(Textbook Pages: Pages 25 – 27 and 53 – 55)

- A) Complete the mathematical rule of exponents.
- B) Complete the corresponding numerical problem.
- C) **Describe** in your own words the rule of exponents.

1) PRODUCT RULE --- *Multiplying exponential expressions with the same base*

A) $a^m \cdot a^n = \underline{a^{m+n}}$

B) $x^{12} \cdot x^5 = \underline{x^{17}}$

C) When you multiply exponential expressions with the same base, add the exponents.

2) QUOTIENT RULE --- *Dividing exponential expressions with the same base*

A) $\frac{a^m}{a^n} \ a \neq 0 = \underline{a^{m-n}}$

B) $\frac{x^{12}}{x^5} \ x \neq 0 = \underline{x^7}$

C) When you divide exponential expressions with the same base, subtract the exponents.

3) POWER RULE --- *Raising an exponential expression to a power*

A) $(a^m)^n = \underline{a^{m \cdot n}}$

B) $(x^{12})^6 = \underline{x^{72}}$

C) When you raise an exponential expression to a power, multiply the exponents.

4) PRODUCT TO POWER RULE --- *Raising a product to a power*

A) $(ab)^m = \underline{a^m b^m}$

B) $(7x)^2 = \underline{7^2 x^2 = 49x^2}$

C) When you raise a product to a power, distribute the exponents.

5) QUOTIENT TO POWER RULE --- *Raising a quotient to a power*

A) $\left(\frac{a}{b}\right)^m \ b \neq 0 = \underline{\frac{a^m}{b^m}}$

B) $\left(\frac{x}{3}\right)^3 = \underline{\frac{x^3}{3^3} = \frac{x^3}{27}}$

D) When you raise a quotient to a power, distribute the exponents.

6) ZERO POWER RULE --- Raising an expression to the zero power

A) $a^0 \quad a \neq 0 = \underline{1}$

B) $7^0 = \underline{1}$

C) Anything, except 0, to the 0 power is 1.

7) NEGATIVE EXPONENT RULE --- Raising an expression to a negative power

A) $a^{-m} \quad a \neq 0 = \underline{\frac{1}{a^m}}$

B) $x^{-12} \quad x \neq 0 = \underline{\frac{1}{x^{12}}}$

C) A negative exponent indicates a reciprocal.

8) Adding exponential expressions with the same base (Hint: Do part B first)

A) $a^m + a^n = \underline{a^m + a^n}$

B) $2^2 + 2^3 = \underline{4 + 8 = 12}$

C) The addition of exponential expressions with the same base, cannot be simplified with exponents.

9) Raising a negative quantity to a power ---

Indicate the multiplication problem and the answer for each of the following

A) $(-5)^2 = \underline{(-5)(-5)} = \underline{25}$

C) $-5^2 = \underline{-(5 \cdot 5)} = \underline{-25}$

B) $(-2)^3 = \underline{(-2) \cdot (-2) \cdot (-2)} = \underline{-8}$

D) $-2^3 = \underline{-(2 \cdot 2 \cdot 2)} = \underline{-8}$

With reference to the above problems, explain the effect parentheses have when evaluating powers.

If the negative is included in the parentheses it is multiplied the number of times indicated by the exponent.

D) Exercises: Page 33 #1 – 12

1. True

2. True

3. False (7^6)

4. False (1)

5. $x^2 + x^2 = 1x^2 + 1x^2 = (1 + 1)x^2 = 2x^2$ --- You add like terms by adding the coefficients and keeping the like term
You add exponents when you multiply exponential expressions with the same base: $x^2 \cdot x^2 = x^4$

6. Exponents can only be distributed over multiplication and division, NOT over addition.

$(x + y)^2 = (x + y)(x + y) = x^2 + 2xy + y^2$

7. $(2^2)^5 = 2^{10}$

8. $(6^4)^3 = 6^{12}$

9. $(2x^5y^4)^3 = 8x^{15}y^{12}$

10. $(-4m^3n^9)^2 = 16m^6n^{18}$

11. $-\left(\frac{p^4}{q}\right)^2 = -\frac{p^8}{q^2}$

12. $\left(\frac{r^8}{s^2}\right)^3 = \frac{r^{24}}{s^6}$

ALGEBRA REVIEW: Radicals

(Textbook Pages: Pages 68 – 72)

A) Simplify the following radicals. Show the prime factorization for each radicand as indicated by the examples.

Example 1: $\sqrt{9504}$

$$\begin{aligned} &\sqrt{2^5 \cdot 3^3 \cdot 11} \\ &2^2 \cdot 3 \sqrt{2 \cdot 3 \cdot 11} = 12\sqrt{66} \end{aligned}$$

Example 2: $\sqrt{3472875}$

$$\begin{aligned} &\sqrt{3^4 \cdot 5^3 \cdot 7^3} \\ &3^2 \cdot 5 \cdot 7 \sqrt{5 \cdot 7} = 315\sqrt{35} \end{aligned}$$

$$\begin{aligned} 1. \quad &\sqrt{1584} \\ &\sqrt{2^4 \cdot 3^2 \cdot 11} \\ &2^2 \cdot 3 \sqrt{11} = 12\sqrt{11} \end{aligned}$$

$$\begin{aligned} 2. \quad &\sqrt{380} \\ &\sqrt{2^2 \cdot 5 \cdot 19} \\ &2\sqrt{5 \cdot 19} = 2\sqrt{95} \end{aligned}$$

$$\begin{aligned} 3. \quad &\sqrt{1452} \\ &\sqrt{2^2 \cdot 3 \cdot 11^2} \\ &2 \cdot 11 \sqrt{3} = 22\sqrt{3} \end{aligned}$$

$$\begin{aligned} 4. \quad &\sqrt{2645} \\ &\sqrt{5 \cdot 23^2} \\ &23\sqrt{5} \end{aligned}$$

$$\begin{aligned} 5. \quad &\sqrt{11907} \\ &\sqrt{3^5 \cdot 7^2} \\ &3^2 \cdot 7 \sqrt{3} \\ &63\sqrt{3} \end{aligned}$$

$$\begin{aligned} 6. \quad &\sqrt{4275} \\ &\sqrt{3^2 \cdot 5^2 \cdot 19} \\ &3 \cdot 5 \sqrt{19} \\ &15\sqrt{19} \end{aligned}$$

B) Perform the indicated operations. Show all work. Do NOT use a calculator.

An example problem for each operation is given.

Addition/Subtraction Example: $\sqrt{48} - 5\sqrt{27}$

$$4\sqrt{3} - 5 \cdot 3\sqrt{3} = 4\sqrt{3} - 15\sqrt{3} = -11\sqrt{3}$$

$$\begin{aligned} 1. \quad &\sqrt{52} + \sqrt{117} \\ &2\sqrt{13} + 3\sqrt{13} \\ &5\sqrt{13} \end{aligned}$$

$$\begin{aligned} 2. \quad &\sqrt{175} - \sqrt{252} \\ &5\sqrt{7} - 6\sqrt{7} \\ &-\sqrt{7} \end{aligned}$$

$$\begin{aligned} 3. \quad &5\sqrt{60} + 2\sqrt{135} \\ &5 \cdot 2\sqrt{15} + 2 \cdot 3\sqrt{15} \\ &10\sqrt{15} + 6\sqrt{15} \\ &16\sqrt{15} \end{aligned}$$

$$\begin{aligned} 4. \quad &2\sqrt{80} - 3\sqrt{45} + 3\sqrt{245} \\ &2 \cdot 4\sqrt{5} - 3 \cdot 3\sqrt{5} + 3 \cdot 7\sqrt{5} \\ &8\sqrt{5} - 9\sqrt{5} + 21\sqrt{5} \\ &20\sqrt{5} \end{aligned}$$

Multiplication Example: $\sqrt{12} \cdot \sqrt{54}$ (Note: Do NOT use a calculator; Use FACTORS)

$$\sqrt{12 \cdot 54} = \sqrt{(2^2 \cdot 3) \cdot (2 \cdot 3^3)} = \sqrt{2^3 \cdot 3^4} = 2 \cdot 3^2 \sqrt{2} = 18\sqrt{2}$$

$$\begin{aligned} 5. \quad &\sqrt{72} \cdot \sqrt{42} \\ &\sqrt{(2^3 \cdot 3^2) \cdot (2 \cdot 3 \cdot 7)} \\ &\sqrt{2^4 \cdot 3^3 \cdot 7} = 2^2 \cdot 3 \sqrt{3 \cdot 7} \\ &12\sqrt{21} \end{aligned}$$

$$\begin{aligned} 6. \quad &\sqrt{92} \cdot \sqrt{115} \\ &\sqrt{(2^2 \cdot 23) \cdot (5 \cdot 23)} \\ &\sqrt{2^2 \cdot 5 \cdot 23^2} = 2 \cdot 23 \sqrt{5} \\ &46\sqrt{5} \end{aligned}$$

$$7. \quad (2\sqrt{40})(3\sqrt{60})$$

$$8. \quad \sqrt{288}(3\sqrt{108})$$

$$2 \cdot 3 \sqrt{(2^3 \cdot 5) \cdot (2^2 \cdot 3 \cdot 5)}$$

$$2 \cdot 3 \sqrt{2^5 \cdot 3 \cdot 5^2} = 2^3 \cdot 3 \cdot 5 \sqrt{2 \cdot 3}$$

$$120\sqrt{6}$$

$$3 \sqrt{(2^5 \cdot 3^2) \cdot (2^2 \cdot 3^3)}$$

$$3 \sqrt{2^7 \cdot 3^5} = 2^3 \cdot 3^3 \sqrt{2 \cdot 3}$$

$$216\sqrt{6}$$

Division Example: $\sqrt{\frac{5}{54}} = \frac{\sqrt{5}}{\sqrt{54}}$

NOTE:
If possible reduce
before simplifying
or rationalizing

$$\frac{\sqrt{5}}{\sqrt{2 \cdot 3^3}}$$

Rationalize the denominator:

$$\left(\frac{\sqrt{5}}{\sqrt{2 \cdot 3^3}} \right) \frac{\sqrt{6}}{\sqrt{2 \cdot 3}}$$

$$\left(\frac{\sqrt{30}}{\sqrt{2^2 \cdot 3^4}} \right) = \frac{\sqrt{30}}{2 \cdot 3^2} = \frac{\sqrt{30}}{18}$$

9. $\frac{\sqrt{75}}{\sqrt{28}}$

$$\frac{\sqrt{3 \cdot 5^2}}{\sqrt{2^2 \cdot 7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{3 \cdot 5^2 \cdot 7}}{\sqrt{2^2 \cdot 7^2}}$$

$$\frac{5\sqrt{3 \cdot 7}}{2 \cdot 7} = \frac{5\sqrt{21}}{14}$$

10. $\frac{3\sqrt{11}}{\sqrt{24}}$

$$\frac{3\sqrt{11}}{\sqrt{2^3 \cdot 3}} \cdot \frac{\sqrt{2 \cdot 3}}{\sqrt{2 \cdot 3}} = \frac{3\sqrt{2 \cdot 3 \cdot 11}}{\sqrt{2^4 \cdot 3^2}}$$

$$\frac{3\sqrt{66}}{2^2 \cdot 3} = \frac{\sqrt{66}}{4}$$

11. $\frac{\sqrt{125}}{\sqrt{108}}$

$$\frac{\sqrt{5^3}}{\sqrt{2^2 \cdot 3^3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3 \cdot 5^3}}{\sqrt{2^2 \cdot 3^4}}$$

$$\frac{5\sqrt{3 \cdot 5}}{2 \cdot 3^2} = \frac{5\sqrt{15}}{18}$$

12. $\frac{3\sqrt{90}}{2\sqrt{126}}$

$$\frac{3\sqrt{2 \cdot 3^2 \cdot 5}}{2\sqrt{2 \cdot 3^2 \cdot 7}} = \frac{3\sqrt{5}}{2\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{3\sqrt{5 \cdot 7}}{2\sqrt{7^2}}$$

$$\frac{3\sqrt{35}}{2 \cdot 7} = \frac{3\sqrt{35}}{14}$$

ALGEBRA REVIEW: Complex Numbers

(Textbook Pages: Pages 103-110)

A) Define each of the following:

1) $i = \sqrt{-1}$, an imaginary unit

3) Pure imaginary Number –a complex number in the form of $a+bi$, where $a = 0$. (no real part)

2) $\sqrt{-a}$ if $a > 0$
then $\sqrt{-a} = i\sqrt{a}$

4) Complex Number
Numbers of the form $a+bi$, where a and b are real numbers

B) Write each of the following in standard form:

1) $\sqrt{-81}$
 $0 + 9i$

2) $-\sqrt{-14}$
 $0 - i\sqrt{14}$

3) $-3 - \sqrt{-12}$
 $-3 - 2i\sqrt{3}$

4) $\sqrt{-196}$
 $0 + 14i$

5) $\sqrt{-50}$
 $0 + 5i\sqrt{2}$

6) $19 + \sqrt{-16}$
 $19 + 4i$

C) Perform the indicated operations. SHOW ALL WORK.

To add and subtract imaginary numbers first simplify each radical; then add or subtract by combining like terms.

1) $3\sqrt{-64} - 2\sqrt{-36}$
 $3 \cdot 8i - 2 \cdot 6i$
 $24i - 12i$
 $12i$

2) $2\sqrt{-8} + 9\sqrt{-18}$
 $2 \cdot 2i\sqrt{2} + 9 \cdot 3i\sqrt{2}$
 $4i\sqrt{2} + 27i\sqrt{2}$
 $31i\sqrt{2}$

3) $9\sqrt{-25} + \sqrt{-100} - 4\sqrt{-64}$
 $9 \cdot 5i + 10i - 4 \cdot 8i$
 $45i + 10i - 32i$
 $23i$

4) $7\sqrt{-80} - 6\sqrt{-20}$
 $7 \cdot 4i\sqrt{5} - 6 \cdot 2i\sqrt{5}$
 $28i\sqrt{5} - 12i\sqrt{5}$
 $16i\sqrt{5}$

When multiplying radicals with negative radicands you must first change each radical to i -form. Remember, when multiplying radical forms use the factors of the radicand rather than the actual number itself; this will make it easier to simplify the radical.

5) $\sqrt{-27} \cdot \sqrt{-60}$
 $i\sqrt{3^3} \cdot i\sqrt{2^2 \cdot 3 \cdot 5} = i^2 \sqrt{2^2 \cdot 3^4 \cdot 5}$
 $-1 \cdot 2 \cdot 3^2 \sqrt{5} = -18\sqrt{5}$

6) $\sqrt{-54} \cdot \sqrt{2}$
 $i\sqrt{2 \cdot 3^3} \cdot \sqrt{2} = i\sqrt{2^2 \cdot 3^3}$
 $2 \cdot 3 \cdot i\sqrt{3} = 6i\sqrt{3}$

7) $(3\sqrt{-5})(-4\sqrt{-12})$
 $3i\sqrt{5} \cdot -4i\sqrt{2^2 \cdot 3} = 3i \cdot -4i\sqrt{2^2 \cdot 3 \cdot 5}$
 $-12i^2 \cdot 2\sqrt{3 \cdot 5} = 24\sqrt{15}$

8) $\sqrt{-8}(\sqrt{-5} - \sqrt{-6})$
 $i\sqrt{2^3}(i\sqrt{5} - i\sqrt{2 \cdot 3}) = i^2 \sqrt{2^3 \cdot 5} - i^2 \sqrt{2^4 \cdot 3}$
 $-1 \cdot 2\sqrt{2 \cdot 5} - -1 \cdot 2^2 \sqrt{3} = -2\sqrt{10} + 4\sqrt{3}$

9) $(5\sqrt{-7})(2\sqrt{14})$
 $5i\sqrt{7} \cdot 2\sqrt{2 \cdot 7} = 10i\sqrt{2 \cdot 7^2}$
 $10 \cdot 7i\sqrt{2} = 70i\sqrt{2}$

10) $\sqrt{-12}(\sqrt{-4} - \sqrt{2})$
 $2i\sqrt{3}(2i - \sqrt{2}) = 4i^2 \sqrt{3} - 2i\sqrt{2 \cdot 3}$
 $-4\sqrt{3} - 2i\sqrt{6}$

When dividing radicals, reduce first; then rationalize the denominator if necessary.

$$11) \frac{\sqrt{-63}}{\sqrt{15}}$$

$$\frac{i\sqrt{21}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{i\sqrt{105}}{5}$$

$$12) \frac{\sqrt{-90} \cdot \sqrt{-15}}{\sqrt{-25} \cdot \sqrt{-27}}$$

$$\frac{i\sqrt{2 \cdot 3^2 \cdot 5} \cdot i\sqrt{3 \cdot 5}}{i\sqrt{5^2} \cdot i\sqrt{3^3}} = \sqrt{2}$$

$$13) (7 - 3i) + (-4 + 7i) - (6 - 8i) = 7 - 3i - 4 + 7i - 6 + 8i = -3 + 12i$$

To add complex numbers, add like terms.

$$14) (5 - 3i)(2 - 4i) = 10 - 20i - 6i - 12 = -2 - 26i$$

To multiply complex numbers, distribute.

$$15) (7 - 6i)^2 = (7)^2 - 2(7)(6i) + (6i)^2 = 49 - 84i - 36 = 13 - 84i$$

Explain the short cut for squaring a complex number.

Do not merely state the formula --- EXPLAIN IN WORDS:

- 1) Square the first term
- 2) Multiply the two terms and double the result
(the sign is the same as the operation of the complex numbers)
- 3) Square the last term (after simplifying the sign will be the opposite because of i^2)
- 4) Combine the like terms.

E) Exercises page 109 #1-28 Complete these problems in the space provided below.

1) A complex number that has no imaginary part is a real number. $(4 + 0i)$

2) A complex number that has a real part is not a imaginary number. $(4 + 0i)$

3) real # 4) real # 5) imaginary # 6) imaginary # 7) complex # (imaginary) 8) complex # (imaginary)

9) $10i$

10) $13i$

11) $-20i$

12) $-15i$

13) $-i\sqrt{39}$

14) $-i\sqrt{95}$

15) $5 + 2i$

16) $-7 + 10i$

17) $9 - 5i\sqrt{2}$

18) $-11 - 2i\sqrt{6}$

19) $i\sqrt{5} \cdot i\sqrt{5} = i^2 \sqrt{25} = -5$

20) $2i\sqrt{5} \cdot 2i\sqrt{5} = 4i^2 \sqrt{25} = -20$

21) $\sqrt{4} = 2$

22) $\sqrt{10}$

23) $3 + 2i + 4 - 3i = 7 - i$

24) $4 - i + 2 + 5i = 6 + 4i$

25) $-2 + 3i + 4 - 3i = 2 + 0i$

26) $-3 + 5i + 4 - 3i = 1 + 2i$

27) $2 - 5i - 3 - 4i + 2 - i = 1 - 10i$

28) $-4 - i - 2 - 3i - 4 + 5i = -10 + i$

ALGEBRA REVIEW: Polynomials

(Textbook Pages: Pages 25 – 36)

A) Define each of the key terms for polynomials.

- 1) algebraic expression: the adding, subtracting, multiplying, dividing (except by 0) or taking of roots on any combination of variables or constants
- 2) term: the product of a real number and one or more variables raised to powers
- 3) like terms: terms with the same variables each raised to the same powers
- 4) coefficient: the real number of the term
- 5) polynomial: a term of a finite number of terms, with only non-negative integer exponents on the variables
- 6) degree ---
 - a. degree of a **term** containing more than one variable: the sum of the exponents on the variables
 - b. degree of a **polynomial in one variable**: the greatest exponent in the polynomial
 - c. degree of a **polynomial in more than one variable**: the greatest degree of any term appearing in the polynomial
- 7) special classifications of polynomials:
 - a. monomial: a one term polynomial
 - b. binomial: a two term polynomial
 - c. trinomial: a three term polynomial

B) Complete the statement indicating the process used to perform the indicated polynomial operations and complete the given problem.

- 1) When adding and subtracting polynomials: adding or subtracting the coefficients of like terms

$$\begin{aligned}(3x^2 - 2x + 3) + 4(5x^2 - 2x + 4) - (2x^2 + 6x - 7) &= 3x^2 - 2x + 3 + 20x^2 - 8x + 16 - 2x^2 - 6x + 7 \\ &= 21x^2 - 16x + 26\end{aligned}$$

- 2) When multiplying polynomials: multiply each term of the second polynomial by each term of the first and add these products

$$(3x + 5)(2x^2 - 3x + 5) = 6x^3 - 9x^2 + 15x + 10x^2 - 15x + 25 = 6x^3 + x^2 + 25$$

- 3) The shortcut for multiplying the sum and difference of the **same** two terms is:
(State the formula AND explain the process in words without using variables.)

$(a + b)(a - b) = a^2 - b^2$ Just multiply the first terms in each binomial and then the last terms and then put a subtraction sign between them

$$(4x^2y^3 - 9z^9)(4x^2y^3 + 9z^9) = 16x^4y^6 - 81z^{18}$$

- 4) The shortcut for squaring a binomial is:
(State the formula AND explain the process in words without using variables.)

$(a + b)^2 = a^2 + 2ab + b^2$ Square the first term and the last term, then double the product of the two term

$$(4x^2y^3 - 9z^9)^2 = (4x^2y^3)^2 - 2(4x^2y^3)(9z^9) + (9z^9)^2 = 16x^4y^6 - 72x^2y^3z^9 + 81z^{18}$$

C) Exercises: Page 33 #13 – 38 Complete these problems in the space provided below.

13) 11th monomial

23) $x^2 - x + 3$

14) 12th binomial

24) $m^3 - 4m^2 + 10$

15) 6th binomial

25) $9y^2 - 4y + 4$

16) 6th trinomial

26) $5p^2 - 3p - 4$

17) 6th binomial

27) $6m^4 - 2m^3 - 7m^2 - 4m$

18) 7th binomial

28) $-6x^3 - 3x^2 - 4x + 4$

19) 6th trinomial

29) $28r^2 + r - 2$

20) 7th binomial

30) $15m^2 + 2m - 24$

21) not a polynomial

31) $15x^2 - \frac{7}{3}x - \frac{2}{9}$

22) not a polynomial

32) $6m^2 + \frac{1}{4}m - \frac{1}{8}$

33) $12x^5 + 8x^4 - 20x^3 + 4x^2$

ALGEBRA REVIEW: Factoring Polynomials

(Textbook Pages: Pages 37-44)

A) DEFINE "FACTORED COMPLETELY" ---

Writing a polynomial/expression as a **PRODUCT** of **prime** polynomials/expressions.

B) TYPES OF FACTORING

1) COMMON FACTOR

a) Multiply: $3y^3(2x^2 - 3xy + 6y^2) = 6x^2y^3 - 9xy^4 + 18y^5$

Using the distributive property and the above information,

FACTOR: $6x^2y^3 - 9xy^4 + 18y^5 = 3y^3 (2x^2 - 3xy + 6y^2)$

b) Describe how you will determine whether the polynomial can be factored using this method AND how you will complete the factoring.

Characteristics of polynomial: **Each term of the polynomial has a common factor**

To complete the factoring:

1) **Determine the GREATEST COMMON FACTOR for all terms of the polynomial**
Write this as the first factor

2) **Divide each term by the GCF --- the quotient is the other factor**

c) Use this process to factor completely each of the following:

1. $25a^5b^3 - 10a^3b^4 + 5a^2b = 5a^2b (5a^3b^2 - 2ab^3 + 1)$

2. $3x^2yz^2 + 18xyz^3 = 3xyz (xz + 6z^2)$

2) DIFFERENCE OF SQUARES

a) Use the special product to multiply: $(5x + 3y)(5x - 3y) = 25x^2 - 9y^2$

Using this information,

FACTOR: $25x^2 - 9y^2 = (5x + 3y)(5x - 3y)$

b) Describe how you will determine whether the polynomial can be factored using this method AND how you will complete the factoring.

Characteristics of polynomial: **Each term is a perfect square; the operations is subtraction**

To complete the factoring: (Include how many and what type of polynomials the factors will be, as well as an explanation of how to determine each of the terms for these factors?)

Two Binomials

The terms in each binomial are the square roots of the terms in the problem

One binomial is +; the other is -

c) Use this process to factor completely each of the following:

1. $121x^4 - 144y^6 = (11x^2 + 12y^3)(11x^2 - 12y^3)$

2. $1 - 81x^4 = (1 + 9x^2)(1 - 9x^2) = (1 + 9x^2) (1 + 3x)(1 - 3x)$

3) SUM OR DIFFERENCE OF CUBES

- a) Use the special product to multiply: $(5x + 3y)(25x^2 - 15xy + 9y^2) = 125x^3 + 27y^3$

Using this information,

FACTOR: $125x^3 + 27y^3 = (5x + 3y)(25x^2 - 15xy + 9y^2)$

- b) Describe how you will determine whether the polynomial can be factored using this method AND how you will complete the factoring.

Characteristics of polynomial: Each term is a perfect cube; Operation may be + or –

To complete the factoring: (Include how many and what type of polynomials the factors will be, as well as an explanation of how to determine each of the terms for these factors?)

A binomial and a trinomial

Binomial Factor:

The terms in the binomial are the cube roots of the terms in the problem

The operation is the same as the original problem

Trinomial Factor:

The first term is the square of the first term of the binomial factor

The second term is the product of the terms of the binomial factor but the opposite sign

The third terms is the square of the second term in the binomial factor

- c) Use this process to factor completely each of the following:

1. $8x^6 - y^3 = (2x^2 - y)(4x^4 + 2x^3y + y^2)$

2. $64a^9 + 216 = (4a^3 + 6)(16a^6 - 24a^3 + 36)$

4) PERFECT SQUARE TRINOMIALS

- a) Use the special product to multiply: $(5x - 3y)^2 = 25x^2 - 30xy + 9y^2$

Using this information,

FACTOR: $25x^2 - 30xy + 9y^2 = (5x - 3y)^2$

- b) Describe how you will determine whether the polynomial can be factored using this method AND how you will complete the factoring.

Characteristics of polynomial:

Two of the terms are perfect squares (1st and 3rd)

The other term is twice the product of the square roots of the perfect square terms

To complete the factoring: (Include how many and what type of polynomials the factors will be, as well as an explanation of how to determine each of the terms for these factors?)

A binomial squared

The terms of the binomial are the square roots of the perfect square term

The operation is the same as the other term

- c) Use this process to determine which of the following can be factored using this method; factor only those that meet the characteristics:

1. $81x^{12} - 90x^6y^8 + 25y^{16} = (9x^6 - 5y^8)^2$

2. $16x^2 - 24x - 9 = \text{cannot be factored using this method}$

3. $36x^{10} + 84x^5y^{11} + 49y^{22} = (6x^5 + 7y^{11})^2$

4. $25x^2 + 15x + 9 =$ cannot be factored using this method

5) TRINOMIALS

a) Use the special product to multiply: $(5x - 3y)(2x + 7y) = 10x^2 + 29xy - 21y^2$

Using this information,

FACTOR: $10x^2 + 29xy - 21y^2 = (5x - 3y)(2x + 7y)$

- b) Use this process to factor completely each of the following. For EACH problem, show your work or explain the process you used for determining the factors – include specific numbers. Merely checking your solution is not acceptable.

1. $8x^2 + 19x - 15 = (8x - 5)(x + 3)$

2. $6x^2 + 11x + 4 = (3x + 4)(2x + 1)$

3. $6x^2 - 5x - 14 = (6x + 7)(x - 2)$

4. $4x^2 + 19x + 12 = (4x + 3)(x + 4)$

ALGEBRA REVIEW: Solving Linear Equations

(Textbook Pages: Pages 84 - 87)

A. Define the three types of equations. For each type indicate how you recognize it and what type of solution it has. Include the example and solution provided in the textbook for each.

1. Contradiction - **An equation that is false for every value of the variable**
2. Identity – **An equation satisfied by every number that is a meaningful replacement for the variable.**
3. Conditional - **Equations satisfied by some numbers but not by others**

B. When solving linear equations:

1. Eliminate all parentheses by distributing
2. Eliminate all fractions and decimals by multiplying by the lowest common denominator
3. Combine like terms
4. Use the Addition and Multiplication Properties of Equality to isolate the variable.

C. Exercises page 89 #19 - 28

Solve the linear equations using the steps above. Show ALL work. Do NOT use a calculator.

19) $2m - 5 = m + 7$

$$m = 12$$

20) $.01p + 3.1 = 2.03p - 2.96$

$$1p + 310 = 203p - 296$$

$$606 = 202p$$

$$3 = p$$

21) $\frac{5}{6}k - 2k + \frac{1}{3} = \frac{2}{3}$

$$5k - 12k + 2 = 4$$

$$-7k = 2$$

$$k = -\frac{2}{7}$$

22) $\frac{3}{4} + \frac{1}{5}r - \frac{1}{2} = \frac{4}{5}r$

$$15 + 4r - 10 = 16r$$

$$5 = 12r$$

$$\frac{5}{12} = r$$

23) $3r + 2 - 5(r + 1) = 6r + 4$

$$3r + 2 - 5r - 5 = 6r + 4$$

$$-2r - 3 = 6r + 4$$

$$-7 = 8r$$

$$-\frac{7}{8} = r$$

24) $5(a + 3) + 4a - 5 = -(2a - 4)$

$$5a + 15 + 4a - 5 = -2a + 4$$

$$9a + 10 = -2a + 4$$

$$11a = -6$$

$$a = -\frac{6}{11}$$

25) $2[m - (4 + 2m) + 3] = 2m + 2$

$$2[m - 4 - 2m + 3] = 2m + 2$$

$$2[-m - 1] = 2m + 2$$

$$-2m - 2 = 2m + 2$$

$$-4m = 4 \quad \text{----} \quad m = 1$$

$$\begin{aligned}
 26) \quad & 4[2p - (3 - p) + 5] = -7p - 2 \\
 & 4[2p - 3 + p + 5] = -7p - 2 \\
 & 4[3p + 2] = -7p - 2 \\
 & 12p + 8 = -7p - 2 \\
 & 19p = -10 \\
 & p = -\frac{10}{19}
 \end{aligned}$$

$$\begin{aligned}
 27) \quad & \frac{1}{7}(3x - 2) = \frac{1}{5}(x + 2) \\
 & 5(3x - 2) = 7(x + 2) \\
 & 15x - 10 = 7x + 14 \\
 & 8x = 24 \\
 & x = 3
 \end{aligned}$$

$$\begin{aligned}
 28) \quad & \frac{1}{5}(2p + 5) = \frac{1}{3}(p + 2) \\
 & 3(2p + 5) = 5(p + 2) \\
 & 6p + 15 = 5p + 10 \\
 & p = -5
 \end{aligned}$$

ARITHMETIC REVIEW

It is expected you will be able to calculate the following types of arithmetic problems WITHOUT the use of a calculator and without long division or multiplication. Show your work or explain your mental math for each problem.

1. $3,400 \times 6,000$ $20,400,000$

2. 2.041×700 $1,428.7$

3. $15,000 \times .3$ $4,500$

4. $6 \overline{)4598}$ $766\frac{1}{3}$

5. $.4 \overline{)7743}$ $11,495$

6. $\frac{63}{15} \cdot \frac{36}{28} \cdot \frac{10}{18}$ 3

7. $\frac{15}{133} \div \frac{27}{95}$ $\frac{25}{63}$

8. $24 \div 2\frac{2}{5}$ 10

9. $19 - 2\frac{5}{9}$ $16\frac{4}{9}$

10. $3 \cdot 7\frac{3}{8}$ $22\frac{1}{8}$

